St. Peter's Church of England Primary School

"Learning - Achieving - Caring"

Calculation Guidance

Mathematics is essential to everyday life: It is critical to science, technology and engineering, necessary for financial literacy and most forms of employment. At St Peter's, our aim is to provide each child with a high quality mathematics education; a foundation for understanding the world, the ability to reason mathematically, appreciate the beauty and power of mathematics and develop a sense of enjoyment and curiosity about the subject. Children are encouraged to make connections across mathematical ideas to develop fluency, mathematical reasoning and competency in solving increasingly sophisticated problems.

This document should be used alongside the Calculation Progression document and seeks to provide guidance based on educational research conducted by the NCETM and Maths Hubs on a number of key areas. Each area will be discussed in more detail with examples below:

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Developing Fluency

Developing Children's Fluency with Basic Number Facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. A short time everyday should be spent on these basic facts which quickly lead to improved fluency. This can be done using simple whole class chorus chanting. This is not meaningless rote learning; rather, this is an important step to developing conceptual understanding through identifying patterns and relationships between tables (for example, that the products in the 6 x table are double the products in the 3 x tables). This will help children develop a strong sense of number relationships, an important prerequisite for procedural fluency.

In KS1, children will learn addition facts to 10 (see diagram below) and in KS2 multiplication tables will be learn to automaticity to avoid cognitive overload in the working memory and enable pupils to focus on new concepts.

Adding 1 and 2 | Bonds to 10 | Adding 10 | Bridging/ compensating

Doubles | Adding 0 | Near doubles

Y1 facts
Y2 facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

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Multiplication tables should be learnt in the following order to provide opportunities to make connections:

×10	×5	×2	×4	×8	×3	×6	×9	×7

Developing Children's Fluency in Mental Calculation

Efficiency in calculation requires having a variety of mental strategies. In particular, the importance of 10 and partitioning numbers to bridge through 10. For example:

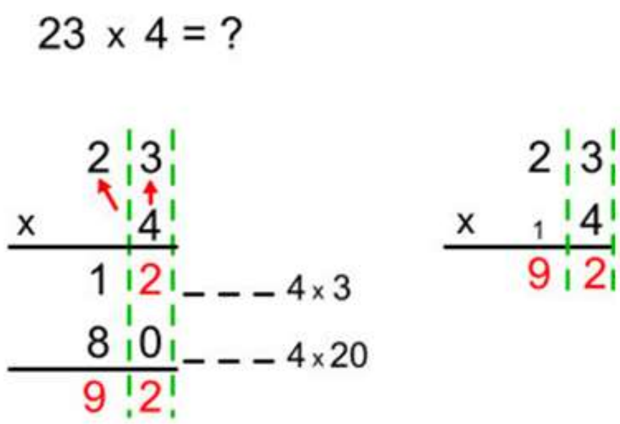
$$9 + 6 = 9 + 1 + 5 = 10 + 5 = 15$$

In order to make this calculation easier, it is helpful to make 10. In Shanghai, the teachers refer to it as "magic 10".

Developing Children's Fluency in the use of Formal Written Methods

Teaching the column method for calculation provides the opportunity to develop both procedural and conceptual fluency. It is important that we ensure that children understand the structure of the mathematics presented in the algorithms, with a particular emphasis on place value. The use of base ten apparatus will support the development of fluency and understanding.

Informal methods of recording calculations are an important stage to help children develop fluency with formal methods of recording. These however should only be used for a short period of times, to help children understand the internal logic of formal methods of recording calculations. They are stepping stones to formal written methods. Here is an example from a Shanghai textbook:



Stepping stones to formal written methods

Developing Children's Understanding of the = Symbol

The symbol = is an assertion of equivalence. If we write:

3 + 4 = 6 + 1

Then we are saying that what is on the left of the = symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used as:

3 + 4 =
5 x 7 =
16 – 9 =

If children only think of = as meaning "work out the answer to this calculation" then they are likely to get confused by empty box questions such as:

3 + ☐ = 8

One way to model equivalence such as 2 + 3 = 5 is to use balance scales.

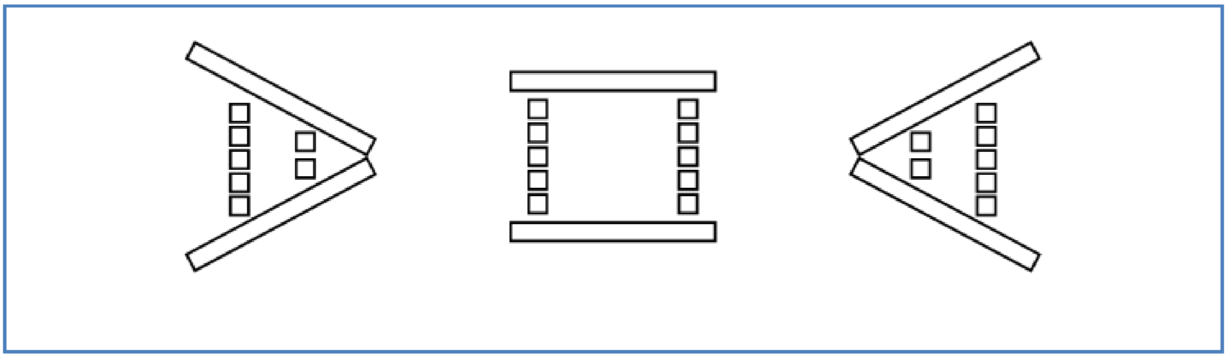
Later they are likely to struggle with even simple algebraic equations, such as:

$$3y = 18$$

The position of the = symbol should be varied from early on in KS1 and should include empty box problems to deepen children's understanding of the = symbol.

Teaching Inequality alongside Teaching Equality

To help young children develop their understanding of equality, they also need to develop understanding of inequality. One way to introduce the < and > signs is to use rods and cubes to make a concrete and visual representation such as:



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This shows that 5 is greater than 2 (5 > 2), 5 is equal to 5 (5 = 5) and 2 is less than 5 (2 < 5).

Incorporating both equality and inequality into examples and exercises can help children develop their conceptual understanding. For example, in this empty box problem children have to decide whether the missing symbol is < = or >.

5 + 7 □ 5 + 6

Balance scales can also be used to represent inequality.

An activity like this also encourages children to develop their mathematical reasoning: "I know 7 is greater than 6, so 5 plus 7 must be greater than 5 plus 6."

Asking children to decide if a number sentence is true or false also helps develop mathematical reasoning. For example, in discussing this statement:

$$4 + 6 + 8 > 3 + 7 + 9$$

a child might reason that "4 plus 6 and 3 plus 7 are both equal to 10. But 8 is less than 9. Therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more than 3 + 7 + 9."

In both these examples the numbers have been deliberately chosen to allow children to establish the answer without actually needing to do the computation. This emphasises further the importance of mathematical reasoning.

Important Strategies

Don't Count, Calculate

Young children benefit from being helped at an early stage to start calculating, rather than relying on 'counting on' as a way of calculating. For example, with a sum such as:

4 + 7 =

Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.

Looking for Patterns and Making Connections

Understanding does not happen automatically, children need to reason by and with themselves and make their own connections supported by the use of visual representations and concrete resources. From Year 1 children should be taught to look for patterns and connections in mathematics. The question: "What's the same and what's different?" should be used frequently to make comparisons. For example: "*What's the same and what's different between the 2 times tables and the 4 times tables?"*

Intelligent Practice

Children should engage in a significant amount of practice of mathematics through class exercises and homework in KS2. However, *in designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity* (Gu, 1991). In China, the practice children engage in provides the opportunity to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections made improve their fluency.

For example:

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

Empty Box Problems

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as:

3 + ☐ = 8

3 + ☐ = 9

3 + ☐ = 10

3 + ☐ = 11

Helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers.

This sequence of examples does the same at a deeper level:

3 × □ + 2 = 20

3 × □ + 2 = 23

3 × □ + 2 = 26

3 × □ + 2 = 29

3 × □ + 2 = 35

Children should also be given examples when the empty box represents the operation, for example:

4 × 5 = 10 ○ 10

6 ○ 5 = 15 + 15

6 ○ 5 = 20 ○ 10

8 ○ 5 = 20 ○ 20

8 ○ 5 = 60 ○ 20

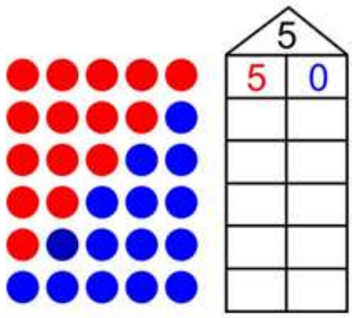
> A circular box ○ should be used to represent the operation and a rectangular box □ used to represent a number.

These examples also illustrate careful use of variation to help develop both procedural and conceptual fluency.

Mathematical Structures

Exposing Mathematical Structures and Working Systematically

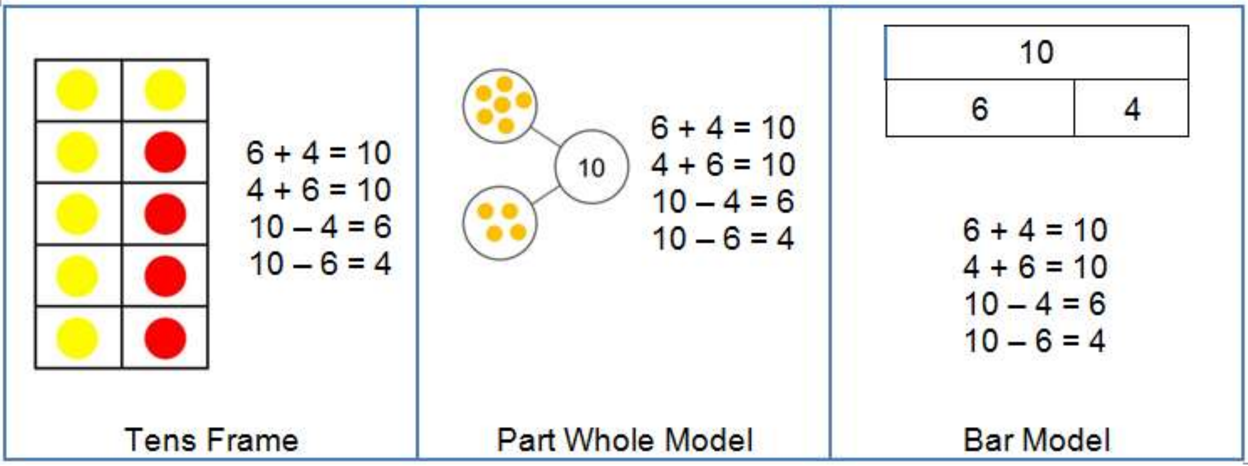
Developing instant recall alongside conceptual understanding of number bonds to 10 is important. This can be supported through the use of images such as in the example illustrated below:



Shanghai Textbook Grade 1 (aged 6/7)

The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.

Using other structured models such as tens frames, part whole models and bar models helps children to reason about mathematical relationships (for further examples see Calculation Progression Document).



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Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question, "What's the same and what's different?" has the potential for children to draw out the connections.

Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change.

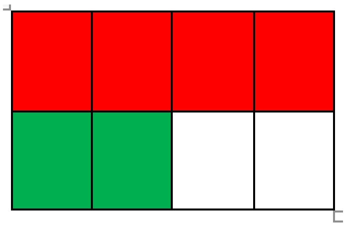
For example:

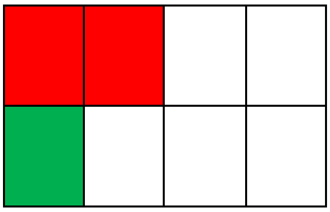
10	247	6.2
6 | 4	173 | 74	3.4 | 2.8
6 + 4 = 10	173 + 174 = 247	3.4 + 2.8 = 6.2
4 + 6 = 10	74 + 173 = 247	2.8 + 3.4 = 6.2
10 – 6 = 4	247 – 173 = 74	6.2 – 3.4 = 2.8
10 – 4 = 6	247 – 74 = 173	6.2 – 2.8 = 3.4

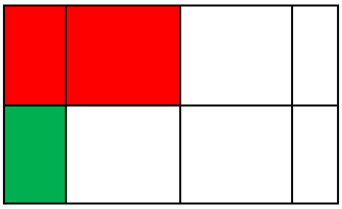
Moving Between the Concrete and the Abstract

Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of and develop fluency in the use of abstract symbols.

For example, in a lesson about addition of fractions children could be asked to draw a picture to represent the sum $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$. Alternatively, or in a subsequent lesson, they could be asked to discuss which of the three visual images correctly represents the sum and to explain their reasoning:







Contextualising the Mathematics

It is important for children to be able to contextualise maths. Maths No Problem textbooks use this concept particularly well and use In Focus tasks to provide a context to learn in. A KS1 lesson about addition and subtraction could start with this contextual story:

"There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are on the bus?"

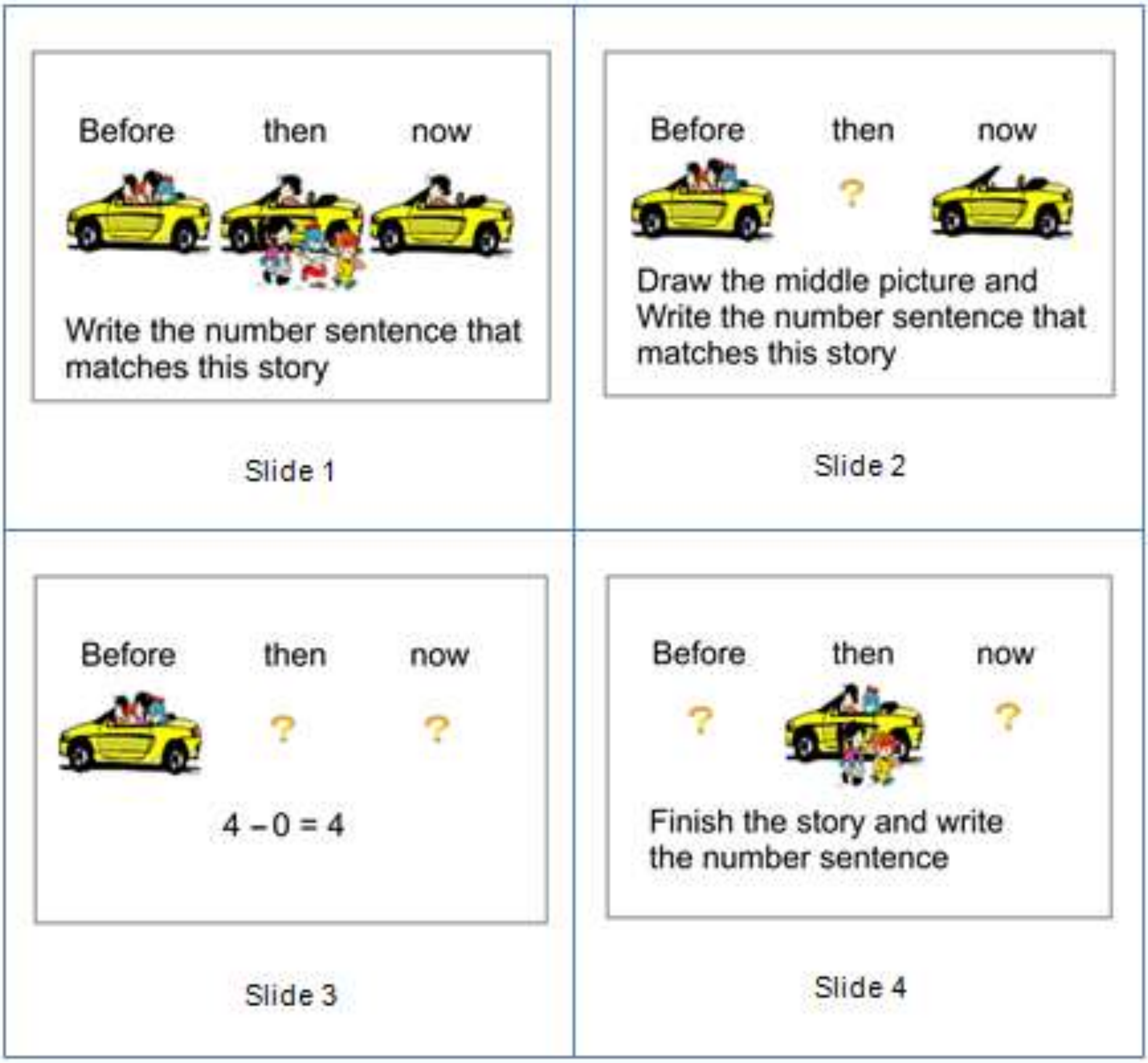
This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story. For example, if the children are thinking about this calculation:

14 – 8

Then the children should be asked:

"What does the 14 mean? What does the 8 mean?" expecting the children to answer: *"There were 14 on the bus, and 8 is the number who got off."*

Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the depth of conceptual understanding and their ability to link the concrete and abstract representations of mathematics.



In the example slides above, each example varies. The children would be asked to:

Slide 1: Start with the story (concrete) and write the equation (abstract).

Slide 2: Start with the story (concrete) and complete it. Then write the equation (abstract).
Slide 3: Start with the equation (abstract) and complete the story (concrete).
Slide 4: Start with part of the story, complete two elements of it (concrete with challenge) and then write the equation (abstract).

The children move between the concrete and abstract and back to the concrete, with an increasing level of difficulty.

Using Questioning to Develop Mathematical Reasoning

Teachers' questions in mathematics lessons should be moving beyond the need to find out whether children can give the right answer to a calculation or problem. But in order to develop children's conceptual understanding and fluency there needs to be a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.

This can be done simply by asking children to explain how they worked out a calculation or solved a problem and to compare and contrast different methods that are described. Once this is embedded in teaching, children quickly come to expect that they need to explain and justify their mathematical reasoning and they soon start to do so automatically and enthusiastically. Some calculation strategies are more efficient and when children's thinking is scaffolded and guided to the most efficient methods, whilst at the same time valuing their own ideas.

Rich questioning strategies include:

- *What's the same, what's different?*

In this sequence of equations, that stays the same each time and what's different?

23 + 10	23 + 20	23 + 30	23 + 40

Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate answers.

- *Odd one out*

Which is the odd one out in this list of numbers? 24, 15, 16 and 22?

This encourages children to apply their existing conceptual understanding. Possible answers could be:

"15 is the odd one out because it's the only off number in the list."
"16 is the odd one out because it's the only square number in the list."
"22 is the odd one out because it's the only number in the list with exactly four factors."

If children are asked to identify an 'odd one out' in this list of products:

24 × 3   36 × 4   13 × 5   32 × 2

They might suggest:
"36 x 4 is the only product whose answer is greater than 100."
"13 x 5 is the product whose answer is an odd number."

- *Here's the answer. What could the question have been?*

Children are asked to suggest possible questions that have a given answer. For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

- *Identify the correct question*

Here children are required to select the correct question:

A 3.5m plank of wood weighs 4.2 kg. The calculation was:

3.5 ÷ 4.2

Was the question:
a. How heavy is 1m of wood?
b. How long is 1kg of wood?

- *True or false*

Children are given a series of equations and are asked whether they are true or false:

$4 \times 6 = 23 \quad 4 \times 6 = 6 \times 4 \quad 12 \div 2 = 24 \div 4 \quad 12 \times 2 = 24 \times 4$

Children are expected to reason about the relationship within the calculation rather than calculate.

- *Greater than, less than or equal to >, < or =*

$3.4 \times 1.2 \bigcirc 3.4 \quad 5.76 \bigcirc 5.76 \div 0.4 \quad 4.69 \times 0.1 \bigcirc 4.69 \div 10$

These types of questions are further examples of intelligent practice where conceptual understanding is developed alongside the development of procedural fluency. They also give advanced learners the opportunity to apply their understanding in more complex ways.

Expect Children to use Correct Mathematical Terminology and to Express their Reasoning in Complete Sentences

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences.

I say, you say, you say, we all say

This technique enables the teacher to provide a sentence stem for the children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. For example:

"If the rectangle is the whole, the shaded part is one third of the whole."

Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repetition also helps us to embed key conceptual knowledge.

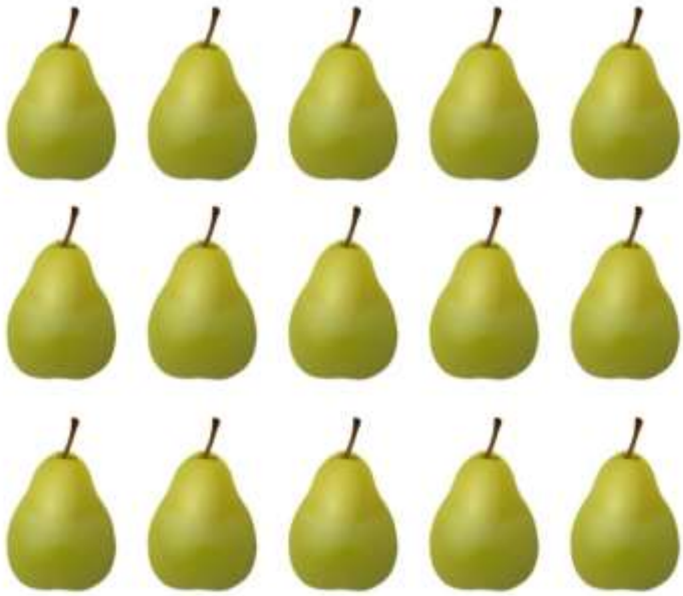
Another example is where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same. For example:

There are 12 **stars**. $\frac{1}{3}$ of the **stars** is equal to **4 stars**

Children use the same sentence stem to express other relationships. For example:

There are 12 **stars**. $\frac{1}{4}$ of the **stars** is equal to **3 stars**
There are 12 **stars**. $\frac{1}{2}$ of the **stars** is equal to **6 stars**

Similarly:



There are **15 pears**. $\frac{1}{3}$ of the **pears** is equal to **5 pears**
There are **15 pears**. $\frac{1}{5}$ of the **pears** is equal to **3 pears**

When talking about fractions it is important to make reference to the whole and the part of the whole in the same sentence. The above examples help children to get into the habit of doing so. Another example is where a mathematical generalisation or “rule” emerges with a lesson. For example:

"When adding 10 to a number, the ones digit stays the same."

This is repeated in chorus using the same sentence, which helps to embed the concept.

Identifying Difficulty Points

Difficult points need to be identified and anticipated when lessons are being designed and these need to be an explicit part of the teaching, rather than the teacher just responding to children’s difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

A visualiser or the airdrop function on an iPad are valuable resources since they allow us to quickly share a child’s thinking with the whole class.

Planning and Assessment Materials

Successful teaching for mastery depends to a large degree on a teacher’s subject knowledge, as well as their understanding of the learning steps required, and the order of those steps. Teaching based on knowledge of mathematical structures and relationships gives pupils the best chance of building deep and secure mathematical understanding. To that end, the NCETM has produced new materials that are designed to assist teachers in their own professional development. The curriculum has been split into five areas called – ‘spines’ – which will continue to be released over the next year.

Alongside the Maths No Problem! textbooks, all teachers should build and design mathematics lessons using the new Mastery Professional Development Materials produced by the NCETM.

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Each calculation objective is broken down into small steps (by year group) and includes videos, teacher guides and representations to build lessons from.

Other useful planning resources include:

- White Rose Small Steps Planning Guides (2017-2018)
- NCETM Reasoning Progression Document
- Collins Shanghai Maths Textbooks (particularly good for challenge activities)
- I See Reasoning E-Books by Gareth Metcalfe
- Kangaroo Maths (Glow Maths Hub)

The NCETM Mastery Assessment materials should be used regularly to assess the depth of children's understanding within a particular objective.

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